

Date: 10/26/18

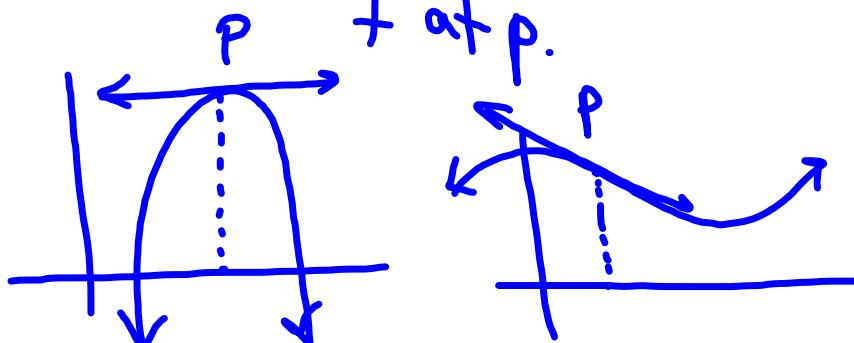
chp: Chp. 3:1 → Derivative of a function

- Obj :
- know the formal definition of a derivative.
 - know the notation for derivatives.
 - Understand the relationships between f & f'
 - Understand the relationship between differentiability & continuity.

The Derivative & the Tangent Line Problem

Anatically $\rightarrow \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Graphically \rightarrow The derivative of a function at pt. p is the slope of the tangent line to the graph of f at p .



Numerically \rightarrow The derivative at a pt. is the limit of the slope of the Secant line.

The derivative at a pt. $x=a$ is found by :

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Or

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Derivative Notation

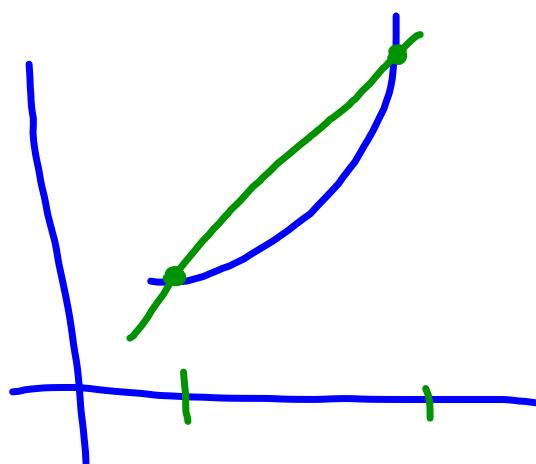
(See p. 101)

1) y' → y prime

2) $\frac{dy}{dx}$ → the derivative of y w/
respect to x.

3) $f'(x)$ → f prime of x

- * A function is differentiable at a given value of x if you can take the derivative of the function at that x value.
- * $f(x)$ is differentiable @ $x=c$ if $f'(c)$ exists.
- * A function that is differentiable @ every pt. of its domain has a derivative.



Ex.1 - Use the definition of a derivative
to find the slope of $f(x) = 3x - 2$
@ $x = -3$

$$f(-3+h) = \frac{3(-3+h)-2}{-9+3h-2} = \frac{-11+3h}{-11+3h}$$

$$y = 3x + b$$

$$-11 = 3(-3) + b$$

$$-2 = b$$

$$f(-3) = 3(-3) - 2 = -11$$

$$\lim_{h \rightarrow 0} \frac{-11+3h+(+11)}{h} = \frac{3h}{h} = 3$$

$$y = 3x - 2$$

Ex.2 - Find the slope of the tangent lines to the graph of $f(x) = x^2 - 5$ at the points $(-2, -1), (1, -4)$

$$f(-2+h) = (-2+h)^2 - 5 \\ h^2 - 4h - 1$$

$$f(-2) = (-2)^2 - 5 = -1$$

$$\lim_{h \rightarrow 0} \frac{h^2 - 4h - 1 + (1)}{h} = \frac{h^2 - 4h}{h} = \frac{h(h-4)}{h} = h-4$$

$$f(-2) = -1 \quad -2 \\ f(x) - f(a)$$

$$\lim_{x \rightarrow -2} \frac{x^2 - 5 + (1)}{x + (-2)} = \frac{x^2 - 4}{x+2} = \frac{(x+2)(x-2)}{x+2}$$

$$\frac{x-2}{-2-2} = -4$$

Ex. 3 - Find the derivative of the function $f(x) = x^3 - 2x$ & evaluate it at $x = 2, 0, -1$

$$\begin{aligned} f(2+h) &= (2+h)^3 - 2(2+h) \\ &= (4+2h+h^2)(2+h) - 4 - 2h \\ &= 8 + 4h + 2h^2 + 4h + 2h + h^3 - 4 - 2h \\ &= h^3 + 2h^2 + 8h + 4 \end{aligned}$$

$$\begin{aligned} f(2) &= 2^3 - 2(2) \\ &= 8 - 4 = 4 \end{aligned}$$

$$\lim_{h \rightarrow 0} \frac{h^3 + 2h^2 + 8h + 4 - 4}{h} = \lim_{h \rightarrow 0} \frac{h(h^2 + 2h + 8)}{h}$$

$$h^2 + 2h + 8 = \textcircled{8}$$

Ex. 4 - Find $f'(x)$ for $f(x) = \sqrt{x}$.

Then find the slopes @ the pts $(4, 2) \& (1, 1)$. What is the behavior @ $(0, 0)$?

$$\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} = \frac{x - a}{(x - a)(\sqrt{x} + \sqrt{a})}$$

$$\lim_{x \rightarrow a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2\sqrt{a}} = m$$

$$f(x) = \sqrt{x}$$

$(4, 2)$

$(1, 1)$ $\lim_{x \rightarrow a}$

$$\begin{array}{c} \text{↗} \\ \text{↗} \\ \text{↗} \\ \text{↗} \\ \text{↗} \end{array}$$

$(0, 0)$

$$f(4) = \frac{1}{2\sqrt{4}}$$

$$= \frac{1}{2\sqrt{4}} = \frac{1}{2(2)} = \frac{1}{4}$$

$$f(0) = \frac{1}{2\sqrt{0}} \quad f(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$\frac{1}{2(0)} = \frac{1}{0} = \text{undefined}$$

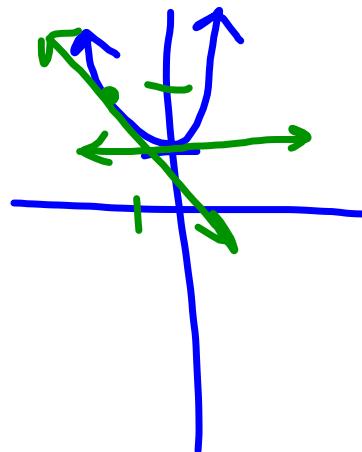
- vert. tangent -

Ex.5 - Find $f'(x)$ of $f(x) = x^2 + 1$.
 Then find the slopes at $(0, 1)$ & $(-1, 2)$

$$f'(x) = 2x$$

$$2(0) = 0$$

$$2(-1) = -2$$



Ex. 6 → Find $f'(x)$ of $f(x) = 2x^2 - 13x + 5$
Then find the slope of
the curve @ $x = 3$. Then find
the equation of the tangent.

Homework:

p. 105 (#1-11 odds, 17-20)